

Problem 1.12

[Difficulty: 3]

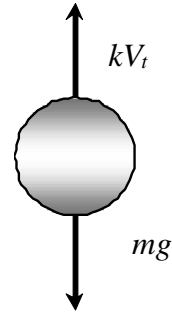
1.12 In a pollution control experiment, minute solid particles (typical mass 1×10^{-13} slug) are dropped in air. The terminal speed of the particles is measured to be 0.2 ft/s. The drag of these particles is given by $F_D = kV$, where V is the instantaneous particle speed. Find the value of the constant k . Find the time required to reach 99 percent of terminal speed.

Given: Data on sphere and terminal speed.

Find: Drag constant k , and time to reach 99% of terminal speed.

Solution: Use given data; integrate equation of motion by separating variables.

The data provided are: $M = 1 \times 10^{-13} \cdot \text{slug}$ $V_t = 0.2 \cdot \frac{\text{ft}}{\text{s}}$



Newton's 2nd law for the general motion is (ignoring buoyancy effects) $M \cdot \frac{dV}{dt} = M \cdot g - k \cdot V$ (1)

Newton's 2nd law for the steady state motion becomes (ignoring buoyancy effects) $M \cdot g = k \cdot V_t$ so $k = \frac{M \cdot g}{V_t}$

$$k = 1 \times 10^{-13} \cdot \text{slug} \times 32.2 \cdot \frac{\text{ft}}{\text{s}^2} \times \frac{\text{s}}{0.2 \cdot \text{ft}} \times \frac{\text{lbf} \cdot \text{s}^2}{\text{slug} \cdot \text{ft}} \quad k = 1.61 \times 10^{-11} \cdot \frac{\text{lbf} \cdot \text{s}}{\text{ft}}$$

To find the time to reach 99% of V_t , we need $V(t)$. From 1, separating variables $\frac{dV}{g - \frac{k}{M} \cdot V} = dt$

Integrating and using limits $t = -\frac{M}{k} \cdot \ln \left(1 - \frac{k}{M \cdot g} \cdot V \right)$

We must evaluate this when $V = 0.99 \cdot V_t$ $V = 0.198 \cdot \frac{\text{ft}}{\text{s}}$

$$t = -1 \times 10^{-13} \cdot \text{slug} \times \frac{\text{ft}}{1.61 \times 10^{-11} \cdot \text{lbf} \cdot \text{s}} \times \frac{\text{lbf} \cdot \text{s}^2}{\text{slug} \cdot \text{ft}} \cdot \ln \left(1 - 1.61 \times 10^{-11} \cdot \frac{\text{lbf} \cdot \text{s}}{\text{ft}} \times \frac{1}{1 \times 10^{-13} \cdot \text{slug}} \times \frac{\text{s}^2}{32.2 \cdot \text{ft}} \times \frac{0.198 \cdot \text{ft}}{\text{s}} \times \frac{\text{slug} \cdot \text{ft}}{\text{lbf} \cdot \text{s}^2} \right)$$

$$t = 0.0286 \text{ s}$$